Magnetic levitation induced by negative permeability

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In this paper we study the interaction between a point magnetic dipole and a semi-infinite meta-material using the method of images. We obtain analytical expressions for the levitation force for an arbitrarily oriented dipole. Surprisingly the maximal levitation force for negative permeability is found to be stronger compared to the case when the dipole is above a superconductor.

Naturally materials have positive permeability, but for the last decade artificial materials with negative permeability (metamaterials) attract growing interest from theoretical and experimental perspectives [1, 2]. In such materials the permeability μ/μ_0 (μ_0-vacuum permeability) is given by

$$\mu/\mu_0 = 1 - \alpha\omega_0^2 / \left(\omega^2 - \omega_0^2\right), \qquad (1)$$

where ω_0 is analog to the plasma frequency (called "magnetic plasma frequency") and α is the so-called filling factor [1, 2]. It is not hard to check that for some frequencies ω the permeability μ is negative.

Most of the papers, related to negative permeability, focus on the optical properties of those metamaterials [3–5]. In the present paper we consider the magnetic forces in the presence of material with negative permeability. We explore theoretically the force acting on a point magnetic dipole placed next to a material with negative permeability. Our examination relies on the Pendry's criterion for validity of magnetostatic limit in the present problem [6]: the wavelength, corresponding to the frequency ω in Eq. (1), should be longer than the distance between the magnetic dipole and the material with negative permeability. Using the method of images [7–9], we show that for negative permeability materials the force between the metamaterial and the magnetic dipole can be attractive or repulsive and can even have a much bigger value compared to the conventional materials.

We consider a point magnetic dipole m in media with permeability μ_1, oriented at angle θ with respect to the axis perpendicular to the plane surface of a medium with permeability μ_2 as shown in Fig. 1. The distance from the dipole to the surface, that separate the two mediums, is d (see Fig. 1). The problem we set is to find how the dipole m interacts with the medium μ_2. The solution is quite straightforward when one uses the method of images [7–9], which states that the magnetic potential in the μ_1 region is a superposition of the dipole fields from m and the image m'

$$m' = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} m, \qquad (2)$$

placed at the same distance d on the other side of the plane separating the two mediums and oriented at the same angle θ with the perpendicular axis to the plane surface (see Fig. 1).
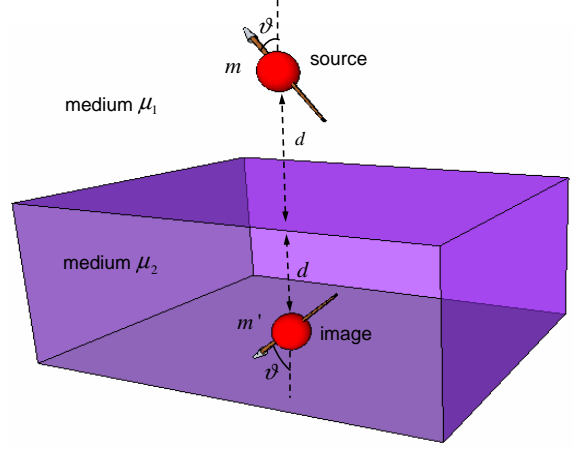

FIG. 1: Magnetic dipole m embedded in a semi-infinite material with permeability μ_1 at a distance d away from a plane interface that separates the two mediums and its dipole image m' symmetrically placed in a semi-infinite material with permeability μ_2.

Using the well-known expression for the potential energy of two real magnetic dipoles m and m' at a distance $\mathbf{r}$ [10, 11]

$$V = \frac{3\mu_1}{4\pi} \left[\frac{\mathbf{m}.\mathbf{m'}}{|r|^3} - \frac{3\left(\mathbf{r} \cdot \mathbf{m}\right)\left(\mathbf{r}.\mathbf{m'}\right)}{|r|^5}\right], \qquad (3)$$

we obtain the following expression for the vertical force acting on a dipole m

$$F = \frac{3\mu_1}{4\pi} \frac{\left(1 + \cos^2\theta\right)mm'}{\left(2d\right)^4} = \frac{3\mu_1 m^2}{4\pi} \frac{\left(1 + \cos^2\theta\right)}{\left(2d\right)^4} \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}, \qquad (4)$$

which force is perpendicular to the surface of a medium with permeability μ_2 and is attractive or repulsive depending on the ratio $\left(\mu_1^2 - \mu_1\mu_2\right) / \left(\mu_1 + \mu_2\right)$. The repulsion force for conventional materials is strongest when the magnetic dipole m is in free space, pointing towards the surface plane and is placed next to a superconductor. This is the Meissner's effect [9, 12–14], which follows formally from Eq. (4) by setting $\theta = 0$, $\mu_1 = \mu_0$ and $\mu_2 = 0$ in this case the vertical lifting force is

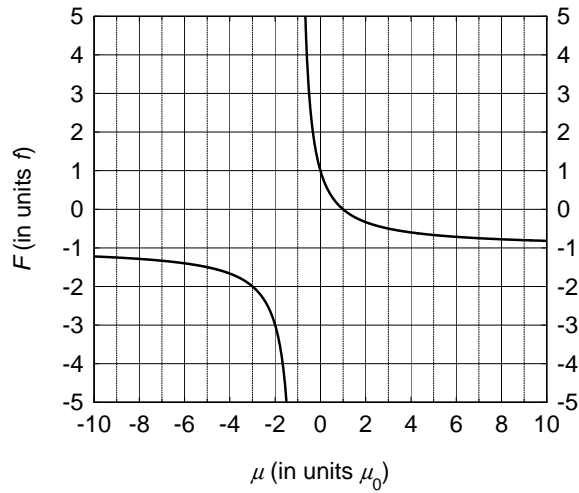$$f = \frac{3\mu_0 m^2}{2\pi \left(2d\right)^4}. \qquad (5)$$

FIG. 2: Maximal vertical force, the case $\theta = 0$ from Eq. (6), as a function of the permeability μ. The force is divided by the maximal force f, which is due to Meissner's effect from Eq. (5).

If we consider the case when the first media is vacuum ($\mu_1 = \mu_0$) while the second media is material with permeability $\mu_2 = \mu$ (material or metamaterial), the force acting on a magnetic dipole is

$$F = \frac{3\mu_0 m^2}{4\pi} \frac{\left(1 + \cos^2\theta\right)}{\left(2d\right)^4} \frac{\mu_0 - \mu}{\mu_0 + \mu}. \tag{6}$$

This force is repulsive when

$$|\mu| < \mu_0 \tag{7}$$

and attractive when

$$|\mu| > \mu_0. \tag{8}$$

Eq. (6) has singularity for $\mu = -\mu_0$. One may think that it leads to an infinite force, but one should note that dispersionless material is an abstraction and a finite positive imaginary component of the permeability always exists [6]. The imaginary component of permeability can no longer be neglected when the difference between the real permeabilities is small, which resolves the infinite force paradox.

Even more interesting than the direction of the force is that the force could be stronger than the one due to the Meissner effect, which can be seen from Fig. 2. For example, when $\mu = -2\mu_0$ the dipole is attracted to the metamaterial with maximal force

$$F = -\frac{9\mu_0 m^2}{2\pi \left(2d\right)^4} = -3f. \tag{9}$$

The last equation and Fig. 2 makes it clear that the levitation force acting on a magnetic dipole can be much stronger than the levitation force on the same dipole above a superconductor. Moreover the levitation in case of metamaterials can be done at room temperature, in contrast to the superconductor levitation case.

The examined force of a point magnetic dipole above a negative permeability has the potential to be not only a curious and intriguing example of what artificial materials can do, but also a useful and effective technique to levitate small objects and therefore to make frictionless devices.

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